### **Partial Differential Equations**

"Beginning Partial Differential Equations"; Peter V. O'Neil

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